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SHORT COMMUNICATION

Comment on "Mixed convection boundary layer flow over a horizontal plate with thermal radiation" by A. Ishak, Heat Mass Transfer, DOI 10.1007/s00231-009-0552-3

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Abstract In a recent paper of Ishak (Heat Mass Transfer, doi:10.1007/s00231-009-0552-3, 2009) the similarity solutions of the title problem have been investigated numerically in some detail. The present note shows, however, that with the aid of a simple rescaling of the Prandtl number, the results reported by Ishak can easily be recovered from the well known solution of the same problem, without the effect of thermal radiation.

The basic reference to the self-similar mixed convection boundary layer flow over a horizontal plate without radiation effects is the classical paper of Schneider [2] published three decades ago (Ref. [5] in [1]). From mathematical point of view, Schneider's pioneering work [2] reduces to the investigation of a two-point boundary value problem for the dimensionless stream function $f = f(\eta)$ and temperature field $\theta = \theta(\eta)$ specified by the coupled differential equations

$$\begin{aligned} &2f''' + ff'' + \lambda\eta\theta = 0,\\ &\frac{2}{Pr}\theta'' + f\theta' + f'\theta = 0 \end{aligned} \tag{1}$$

along with the boundary conditions

$$\begin{aligned} &f(0) = 0,\ f'(0) = 0,\ \theta(0) = 1,\\ &f'(\infty) = 1,\ \theta(\infty) = 0 \end{aligned} \tag{2}$$

In the above equations the primes denote differentiation with respect to the similarity independent variable η, and λ stands for the mixed convection parameter (denoted in [2] by K). After its publication, Schneider's paper [2] has attracted a considerable research interest. This development has been described by Magyari et al. [3] in some detail (Ref. [17] in [1]). As emphasized in [3], the existence of two solution branches of the problem (1), (2), has already been mentioned in the "Note added in proofs" of Schneider's original paper [2].

In the recent paper of Ishak [1], "Schneider's problem" was revisited by including in the energy equation also the contribution of thermal radiation in the *Rosseland approximation*. As it is well known, the only effect of the Rosselend radiation term is that it rescales the thermal diffusion term $\alpha\,\partial^2 T/\partial y^2$ of the energy equation by a constant factor $(1 + N)$, where N denotes the dimensionless radiation parameter (see e.g. Ref. [16] in [1], and further references therein). As a consequence, in the Rosseland approximation adopted in [1], Schneider's second Eq. 1 is simply replaced by

$$\frac{2(1+N)}{Pr}\theta'' + f\,\theta' + f'\theta = 0 \tag{3}$$

(see Eq. 11 of [1]). Now, introducing the notation

$$Pr_0 = \frac{Pr}{1+N} \tag{4}$$

Equation 3 becomes formally identical to the second Eq. 1 with Pr replaced by Pr_0. In other words, the governing equations of the "radiation problem" considered by Ishak [1] become

$$\begin{aligned} &2f''' + ff'' + \lambda\eta\theta = 0,\\ &\frac{2}{Pr_0}\theta' + f\theta = 0 \end{aligned} \tag{5}$$

This analogy has two important consequences. The one is that for a fixed λ and given values of Pr and N, the

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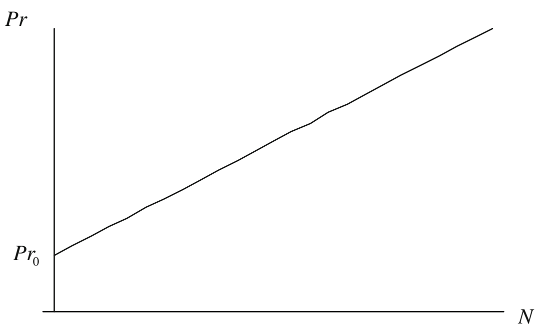

Fig. 1 All the points of the parameter plane (N, Pr) lying on the straight line $Pr = (1 + N)Pr_0$ correspond to one and the same solution of the radiation problem (5), (2), which in turn coincides with the solution of Schneider's problem (1), (2) for $Pr = Pr_0$

solution (f, θ) of the "radiation problem" (5), (2) coincides with the solution of Schneider's problem (1), (2) for the same λ, when Pr replaced in the second Eq. 1 by Pr_0. The second consequence is the converse of the first one. Namely, for a fixed λ and a given value $Pr \equiv Pr_0$ of the Prandtl number, the solution (f, θ) of Schneider's problem (1), (2) is at the same time the solution of the "radiation problem" (5), (2) for the same λ and all values of Pr and N which satisfy the Eq. 4.

Obviously, there are an infinity of such values of the parameter pair (N, Pr) which satisfy Eq. 4 for any given Pr_0. In other words, in the parameter plane (N, Pr), all points lying on the straight line $Pr = (1 + N)Pr_0$ correspond to one and the same solution of the "radiation problem" (5), (2), which in turn coincides with the solution of Schneider's problem (1), (2) for $Pr = Pr_0$ (and the same fixed value of λ). This result is illustrated graphically in Fig. 1. A specific example for this multiplicity can be identified even in Table 1 of [1]. Indeed, the values $(N, Pr) = (0, 0.5)$ and $(N, Pr) = (1, 1)$ correspond according to Eq. 4 to the same value 0.5 of Pr_0. Consequently, the solution of the radiation problem is the same for both $(N, Pr) = (0, 0.5)$ and $(N, Pr) = (1, 1)$, and coincides in turn with the solution of Schneider's problem for $Pr = 0.5$ (and the same λ). On this reason, in all of these three cases the critical value of λ is the same, $\lambda_c = -0.0594$ (see Table 1 of [1]). The small deviation of $\lambda_c = -0.0594$ from the value $\lambda_c = -0.0577$ reported in [2] is a matter of the numerical accuracy only.

Therefore, we may conclude that concerning the solution of the "radiation problem", all the results of Ishak [1] can be recovered without any numerical effort, by a simple transcription of the results reported in the earlier literature of Schneider's mixed convection problem (1), (2). An additional gain which emerges from the present comparative approach is that the "radiation problem" (5), (2) admits the same solution for all the parameter values (N, Pr) which correspond to the same value Pr_0 of the dimensionless group given by Eq. 4.

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